EVALUATION OF A COUPLING HOLE BETWEEN TWO RESONANT CAVITIES

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In microwave technology, resonant cavities may be coupled through a hole in a common wall. The general principles of such coupling are well known.

The present purpose is the presentation of this subject in a unified manner which will emphasize the principles and also will aid in understanding and computing the coupling in a variety of situations. The presentation is quantitative to the extent permitted by some simplifying assumptions.

While most of the subject matter is taken from diverse sources in the literature, the basis for the unified presentation seems to be new. This naturally leads to interesting concepts and relationships that may not have been apparent in the earlier publications.

The field coupling or polarizability of any coupling hole will be expressed in terms of an effective volume. The coupled cavities will likewise be evaluated in terms of an effective volume. Then it becomes possible to formulate, in terms of a volume ratio, the coupling coefficient in any situation where the concept is applicable.

Fig. 1 shows the principle of expressing the amount of coupling in terms of a ratio of two values of effective volume. Two resonant cavities are separated by a common wall, and are coupled by a hole in this wall. As a simplifying assumption, the hole is so located that only one kind of field is effective therein, say the magnetic field. In a manner to be described, with reference to this kind of field, the effective volume of each cavity (V) and the coupling hole (V_c) will be defined and formulated. The coefficient of coupling between the cavities (k), as usually defined in circuit theory, is then expressed in terms of the ratio of these volumes, as indicated. The factor 1/4 is introduced by a concept based on a rule that will be stated.

There is a simple "rule of one-half" which will be presented with reference to a symmetrical small aperture in a thin wall of perfect shielding. The field intensity in the center of the hole is just one-half of that which would have existed at the same location on one side of the wall without the hole. Applying this rule to the field on both sides of the wall, introduces the factor 1/4 in the formula in Fig. 1.

The basis for the effective volume of the hole will be presented by analogy to the effective volume of a thin disc in a uniform field, relying on the principle of duality. This analogy will be supported by an equivalent-network derivation, using circuit theory with the aid of the bisection theorem.

A simple resonant cavity for illustrating these principles is the "circular pillbox" shown in Fig. 2. The field is taken to be two-dimensional as in the circular TM-010 mode of resonance The two cavities with a common side wall for the coupling hole, are coupled by magnetic field in the hole.

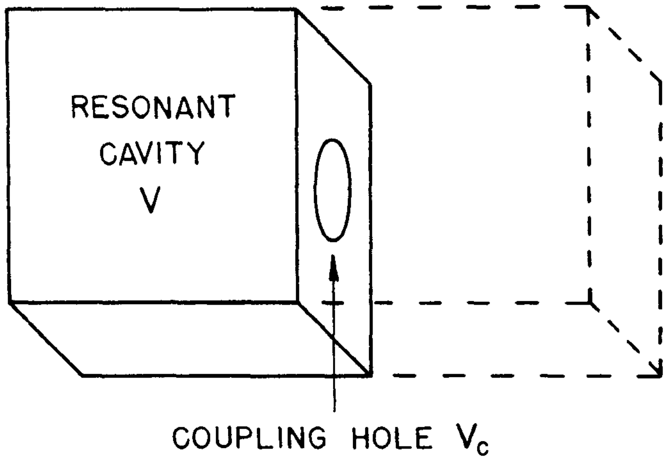
As one step, it is necessary to evaluate the effective area or volume of the resonant cavity. In general terms, the electric field is maximum in the center of the circle and the magnetic field is near maximum at the circular wall. The hole is located in this wall, so the magnetic field intensity (H) at this location (but before the hole is opened) is taken as a reference. The effective area of the resonant cavity is defined as the area (A_m) which would contain the same amount of magnetic energy if filled with uniform intensity equal to the reference value (H). Here it is remarkable that the effective area of the magnetic energy is equal to the area of the cavity; the small density near the center is compensated by an excess of density in an intermediate region just inside the rim. The effective area of the hole being likewise a circle, the result is an extremely simple formula for the coupling coefficient (k_m) as shown.

Going to three-dimensional fields, the simplest case is the magnetic coupling of two cubic resonant cavities by a circular hole, as shown in Fig. 3(a). The resonance is in the pillbox mode, rectangular TM-110, which has a two-dimensional field. However, the coupling hole is circular, which introduces a departure from the two-dimensional field. Using effective volume, instead of area, for both the cavity and the hole (V_m, V_{mc}), the coupling coefficient (k_m) is simply formulated as shown.

Fig. 3(b) has the cavity field reoriented for electric coupling, the hole being located at the maximum electric field intensity (E). It is found that the effective volume of the cavity and that of the hole are both reduced to one-half, so the coupling coefficient remains the same. This is a remarkable coincidence, which might be useful in the design of such cavities for simultaneous utilization of different orthogonal modes at the same frequency.

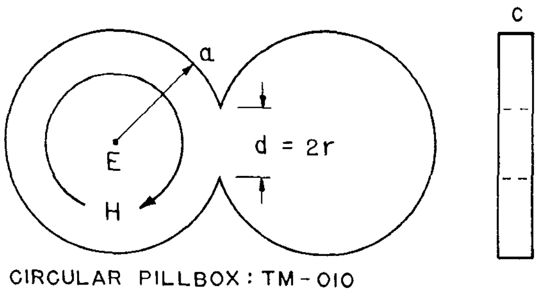
In conclusion, a basis has been presented for evaluating the coupling through an aperture in terms of a ratio of effective area or volume of the aperture relative to that of the adjoining bounded regions. The measure of coupling is the coupling coefficient between two resonant cavities.

These concepts and formulas are intended as an aid in understanding and computing the behavior of coupling apertures between bounded regions of a wave medium.



$$k = \frac{1}{4}\,\frac{V_c}{V}$$

Fig. 1. Coupling coefficient in terms of volume ratio.

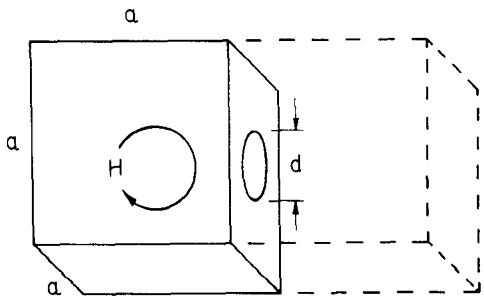


$$A_m = \pi a^2 = \text{circle}(a)$$

$$A_{mc} = \pi r^2 = \text{circle}(r)$$

$$k_m = \frac{A_{mc}}{4A_m} = \frac{1}{4}\left(\frac{r}{a}\right)^2$$

Fig. 2. Circular-pillbox resonant cavities.



$$V_m = \frac{1}{2} a^3 \;; \quad V_{mc} = \frac{2}{3} d^3 = \frac{4}{\pi} \text{ sphere}$$

$$k_m = \frac{V_{mc}}{4 V_m} = \frac{1}{3} \left(\frac{d}{a}\right)^3$$

(a) Magnetic coupling.

Fig. 3. Cubic resonant cavities coupled by a circular hole.
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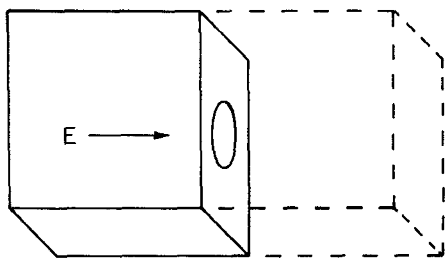

$$V_e = \frac{1}{4} a^3 \;; \quad V_{ec} = \frac{1}{3} d^3 = \frac{2}{\pi} \text{ sphere}$$

$$k_e = \frac{V_{ec}}{4 V_e} = \frac{1}{3} \left(\frac{d}{a}\right)^3 = k_m$$

(b) Electric coupling.

Fig. 3. Cubic resonant cavities coupled by a circular hole.
(b) Electric coupling.

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